day 70

5.1/62) $f(x) = \dfrac{\left(4\sqrt{x} + \frac{6}{\sqrt{x}}\right)}{x^2}$ $F(1) = 4$

$$= \frac{4\sqrt{x}}{x^2} + \frac{\left(\frac{6}{\sqrt{x}}\right)}{x^2} = 4x^{-\frac{3}{2}} + 6x^{-5/2}$$

$$F(x) = \int f(x)\,dx = 4\left(\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}}\right) + 6\left(\frac{x^{-\frac{3}{2}}}{-\frac{3}{2}}\right) + C$$

$$= 4x^{-\frac{1}{2}}(-2) + 6x^{-\frac{3}{2}}\left(-\frac{2}{3}\right) + C$$

$$= -8x^{-\frac{1}{2}} - 4x^{-3/2} + C$$

$$= \frac{-8}{\sqrt{x}} - \frac{4}{(\sqrt{x})^3} + C$$

$$F(x) = \frac{-8}{\sqrt{x}} - \frac{4}{(\sqrt{x})^3} + C$$

$$4 = F(1) = \frac{-8}{\sqrt{1}} - \frac{4}{(\sqrt{1})^3} + C = -8 - 4 + C = -12 + C$$

$$4 = -12 + C$$

so $C = 16$

$$F(x) = \frac{-8}{\sqrt{x}} - \frac{4}{(\sqrt{x})^3} + 16$$

Dec 16-7:18 AM

5.1/42)

day 70

$$\int \frac{\sin\theta - 1}{\cos^2\theta}\,d\theta = \int \frac{\sin x - 1}{\cos^2 x}\,dx$$

$$= \int \frac{\sin x}{\cos^2 x} - \frac{1}{\cos^2 x}\,dx = \int \sec x \tan x - \sec^2 x\,dx$$

$$= \sec x - \tan x + C$$

Dec 16-7:18 AM

day 70

4.7/81)

a) $\lim_{x\to 2} \frac{x-2}{x^2-1} \overset{LH}{=} \lim_{x\to 2} \frac{1}{2x} = 4$

T or F F/N ans = 0 Y or N

b) $\lim_{x\to 0} x\sin x = \lim_{x\to 0} f(x)g(x) =$

$\lim_{x\to 0} f'(x) \lim_{x\to 0} g'(x) =$

$\left(\lim_{x\to 0} 1\right)\left(\lim_{x\to 0} \cos x\right) = 1$

T or F

c) $\lim_{x\to 0^+} x^{\frac{1}{x}}$ is an indeterminate form

FALSE

$\frac{0}{0}, \frac{\infty}{\infty}$
$0\cdot\infty, \infty-\infty$
$1^{\infty}, \infty^0, 0^0$

d) The number raised to any fixed power is 1

Therefore, since $(1+x) \mapsto 1$ as $x \mapsto 0$
$(1+x)^{\frac{1}{x}} \to 1$ as $x \mapsto 0$
$\to e \approx 2.718281828459045\ldots$

Euler

e: the story of a number
Eli Maor

Amir Aczel

Mario Livio

Michio Kaku
Einstein's Cosmos

4.7/81

day 70

e) The functions $\ln x^{100}$ and $\ln x$ have comparable growth rates as $x \to \infty$

$\ln x^{100} = 100 \ln x$

$$\lim_{x\to\infty} \frac{\overset{\ln x^{100} =}{100 \ln x}}{\ln x} = \underline{100}$$

f) The function e^x grows faster than 2^x as $x \to \infty$

$$\lim_{x\to\infty} \frac{e^x}{2^x} = \lim_{x\to\infty} \left(\frac{e}{2}\right)^{\infty} = \infty$$

day 70

5.2/9) obj moving along a line

$$v(t) = 3t^2 + 1 \quad , [0,4]$$

a) 4 subintervals: $[0,1], [1,2], [2,3], [3,4]$
evaluate $v(t)$ at midpt of each interval.

Approximate displacement =

$$(1)\left(v\left(\tfrac{1}{2}\right)\right) + (1)\cdot v\left(\tfrac{3}{2}\right) + (1)v\left(\tfrac{5}{2}\right) + 1\cdot v\left(\tfrac{7}{2}\right)$$

$$= 3\left(\tfrac{1}{2}\right)^2 + 1 + 3\left(\tfrac{3}{2}\right)^2 + 1 + 3\left(\tfrac{5}{2}\right)^2 + 1 + 3\left(\tfrac{7}{2}\right)^2 + 1$$

$$= 3\left(\frac{1^2 + 3^2 + 5^2 + 7^2}{4}\right) + 4 = 3(21) + 4 = 67$$

BUT WAIT!
does this mean displacement $= \int_a^b v(t)\,dt$?

b) $n = 8$:

$$\tfrac{1}{2}\left(3\left(\tfrac{1}{4}\right)^2 + 1\right) + \tfrac{1}{2}\left(3\left(\tfrac{3}{4}\right)^2 + 1\right) + \cdots + \tfrac{1}{2}\left(3\left(\tfrac{15}{4}\right)^2 + 1\right)$$

height (value of function) over each subinterval

width of each rectangle

$$= \frac{1}{2}\left(3\sum_{k=1}^{8}\frac{(2k-1)^2}{16} + 8(1)\right) = 4 + \frac{3}{32}\sum_{k=1}^{8}(2k-1)^2$$

$= \ldots$

5.1/38)

day 70

$$\int \sin 4t - \sin \frac{t}{4}\, dt$$

$$= -\frac{1}{4}\cos(4t) + 4\cos\frac{t}{4} + C$$

figuring

$\frac{d}{dt}\cos(4t) =$

$-\sin(4t) \cdot 4$

in integral talk

$\int (-4)\sin 4t\, dt$

$= \cos(4t)$